Cloud microphysics

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Overview of cloud physics lecture

- Atmospheric thermodynamics
  - gas laws, hydrostatic equation
  - 1st law of thermodynamics
  - moisture parameters
  - adiabatic / pseudoadiabatic processes
  - stability criteria / cloud formation
- Microphysics of warm clouds
  - nucleation of water vapor by condensation
  - growth of cloud droplets in warm clouds (condensation, fall speed of droplets, collection, coalescence)
  - formation of rain, stochastical coalescence
- Microphysics of cold clouds
  - homogeneous nucleation
  - heterogeneous nucleation
  - contact nucleation
  - crystal growth (from water phase, riming, aggregation)
  - formation of precipitation
- Observation of cloud microphysical properties
- Parameterization of clouds in climate and NWP models

Growth rate and size distribution

- growing droplets consume water vapor faster than it is made available by cooling and supersaturation decreases
- haze droplets evaporate, activated droplets continue to grow by condensation

growth rate of water droplet

$$\frac{dr}{dt} = G_l S \frac{1}{r}$$

- smaller droplets grow faster than larger droplets
- sizes of droplets in cloud become increasingly uniform, approach monodisperse distribution

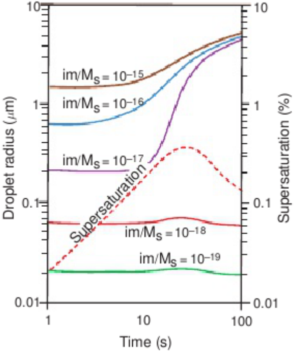
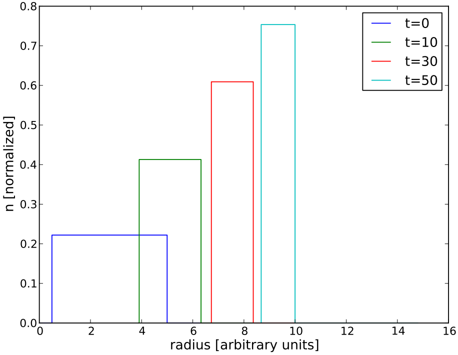


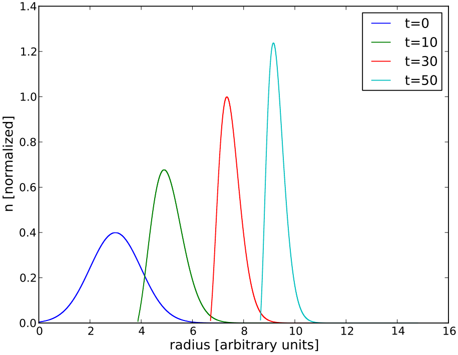
Fig. 6.16 Theoretical computations of the growth of cloud condensation nuclei by condensation in a parcel of air rising with a speed of 60 cm s^{-1}. A total of 500 CCN cm^{-1} was assumed with im/M_s values [see Eq. (6.8)] as indicated. Note how the droplets that have been activated (brown, blue, and purple curves) approach a monodispersed size distribution after just 100 s. The variation with time of the supersaturation of the air parcel is also shown (dashed red line). [Based on data from *J. Meteor.* **6**, 143 (1949).]

Figure from Wallace and Hobbs

Size distribution evolution

$$r = \sqrt{r_0^2 + 2G_l S t}$$





Growth by collection

- growth by condensation alone does not explain formation of larger drops
- other mechanism: growth by collection

Terminal fall speed

$r \lesssim 40\mu$

$$v = \frac{2}{9}\frac{g(\rho-\rho')r^2}{\eta}$$

η – viscosity of air

$40\mu m \lesssim r \lesssim 0.6mm$

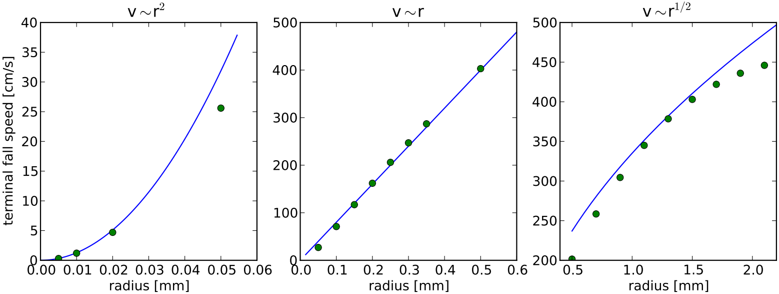
$$v = kr$$

$k=8\cdot10^3 s^{-1}$

$0.6mm \lesssim r \lesssim 2mm$

$$v = kr^{\frac{1}{2}}$$

$k=2.2\cdot10^3\left(\frac{\rho}{\rho'}\right)^{\frac{1}{2}} cms^{-1}$



Larger droplets approach constant value of about 10m/s.

Growth by collection

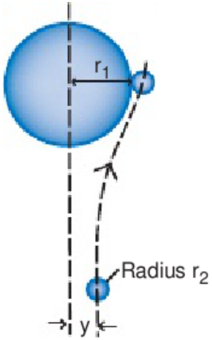


Fig. 6.19 Relative motion of a small droplet with respect to a collector drop. y is the maximum impact parameter for a droplet of radius r_2 with a collector drop of radius r_1.

Figure from Wallace and Hobbs

Collision efficiency

Collision efficiency

$$E = \frac{y^2}{r_1^2 + r_2^2}$$

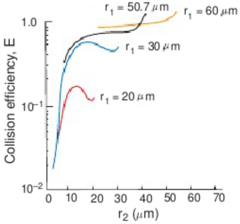


Fig. 6.20 Calculated values of the collision efficiency, E, for collector drops of radius r_1 with droplets of radius r_2. [Adapted from H. R. Pruppacher and J. D. Klett, *Microphysics of Clouds and Precipitation*, Kluwer Academic Pub., 1997, Fig. 14-6, p. 584, Copyright 1997, with kind permission of Springer Science and Business Media. Based on *J. Atmos. Sci.* **30**, 112 (1973).]

Figure from Wallace and Hobbs

- E increases when size of collector drop r_1 increases
- E small for $r_1 < 20\mu$m
- $r_1 \gg r_2$: E small because small droplets follow streamlines around collector drop
- E increases with increasing r_2 until $r_2/r_1 \approx$ 0.6–0.9
- r_2/r_1 >0.6–0.9: E decreases because relative velocity between droplets becomes small
- r_2/r_1 ≈1: strong interaction between droplets, E increases again

Coalescence efficiency

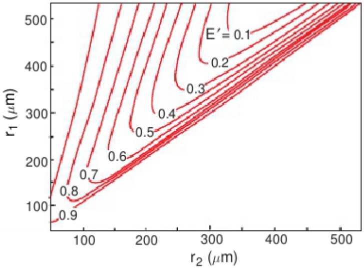


Fig. 6.22 Coalescence efficiencies E' for droplets of radius r_2 with collector drops of radius r_1 based on an empirical fit to laboratory measurements. [Adapted from *J. Atmos. Sci.* **52**, 3985 (1995).]

Figure from Wallace and Hobbs

Coalescence efficiency

E'=fraction of collisions that result in coalescence

- E' large for $r_2 \ll r_1$
- E' initially decreases as r_2 increases
- as r_2 approaches r_1, E' increases sharply

Collection efficiency

collection = collision + coalescence

Collection efficiency

$$E_c = E \cdot E'$$

Continuous collection model

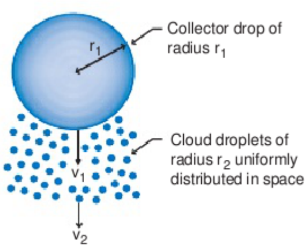


Fig. 6.23 Schematic illustrating the continuous collection model for the growth of a cloud drop by collisions and coalescence.

Figure from Wallace and Hobbs

- collector drop with radius r_1 and terminal velocity v_1
- drop falls in still air through cloud of equal sized droplets with r_2 and v_2
- droplets are uniformly distributed and collected uniformly by all collector drops of a given size

Continuous collection model

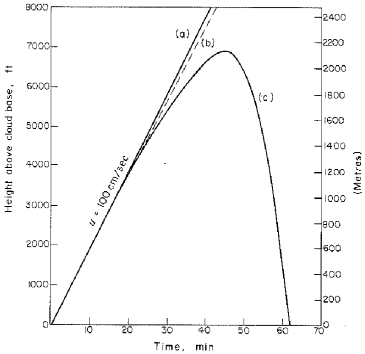


FIG. 7.5. Bowen's calculated trajectories of (a) the air, (b) cloud droplets, initially 10 μm in radius, and (c) drops which have initially twice the mass of the cloud droplets. Updraft speed 1 m/sec, cloud water content $M = 1$ g/m^3. (From Fletcher, 1962.)

Figure from Rogers

$$\frac{dr_1}{dt} \approx \frac{v_1 \omega_l E}{4\rho_l}$$

$$H \approx \frac{4\rho_l}{\omega_l} \int_{r_0}^{r_H} \frac{w - v_1}{v_1 E} dr_1$$

Bowen (1955) assumed E=1.

Continuous collection model (Bowen model)

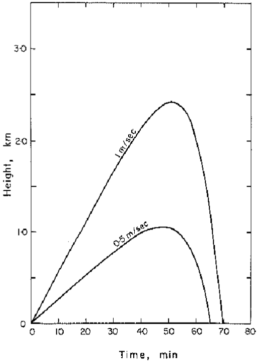


FIG. 7.6. Drop trajectories calculated for the collision efficiencies of Table 7.3 and Fig. 7.4, assuming a coalescence efficiency of unity. Initial drop radius 20 μm. Cloud water content 1 g/m^3; all cloud droplets of 10 μm radius.

Figure from Rogers

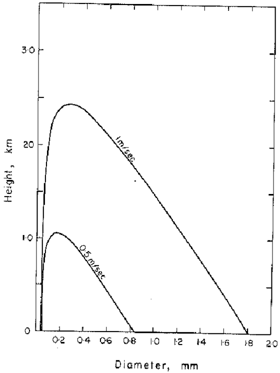


FIG. 7.7. Drop diameters for the trajectories of Fig. 7.6.

Figure from Rogers

Statistical growth: Telford model

- local variations in droplet concentration in clouds

PDF for number of droplets in volume V

$$p(m) = e^{-\bar{n}V} \frac{(\bar{n}V)^m}{m!}$$

Poisson probability law

$\bar{n}$ – average droplet concentration in given size interval
m – number of droplets in V

- "fortunate" drops fall through regions of locally high concentration $\Rightarrow$ experience more than average number of collisions $\Rightarrow$ grow more rapidly
- 1 of 10^5 or 10^6 droplet is sufficient to initiate rain

Statistical growth: Telford model

- Assumptions: r_1=12.6μm, r_2=10μm $\Rightarrow V(r_1) = 2V(r_2)$, E=1
- transformation of bimodal size distribution in broad size distribution happens
  - in most cases within t$<$0.5min
  - in few cases within t$>$30min
- PDF for time required for collector drop to experience given number of collisions (figure)
- most likely case corresponds to continuous collection model
- drops that grow faster than average may account for development of rain

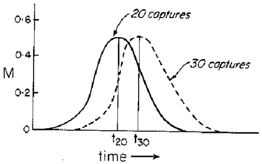


FIG. 7.8. Distribution curves of time required for 20 and 30 collisions of a collector drop (schematic).

Figure from Rogers

Statistical growth: Telford model

- Robertson (1974) expanded Telford model by including realistic E
- E not analytical ⇒Monte Carlo approach
- PDF for time required to experience given number of collisions
- PDFs approach limiting form
- standard deviation σ becomes negligible for r>40μm ⇒corresponds to continuous collection model
- r<20μm⇒E too small although σ large no drizzle or rain formation

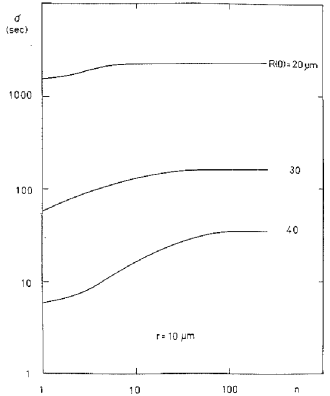


FIG. 7.9. The standard deviation σ of time required to make n captures, for $r = 10\ \mu m$ and $R(0) = 20$, 30, and 40 μm. Cloud liquid water content $M = 1\ g/m^3$. (From Robertson, 1974.)

Figure from Rogers

Gap between condensational and collectional growth

- condensational growth
  - slows appreciably as droplet radius approaches $\sim 10\mu$m
  - tends to produce monodisperse size distribution
  - droplets then have similar fall speeds $\Rightarrow$ collisions become unlikely
- collectional growth
  - conditions: a few reasonably efficient collector drops (i.e. r$> 20\mu$m) cloud deep enough and contains sufficient amount of water
- Question 1: How do the collector drops initially form

Broad size distributions

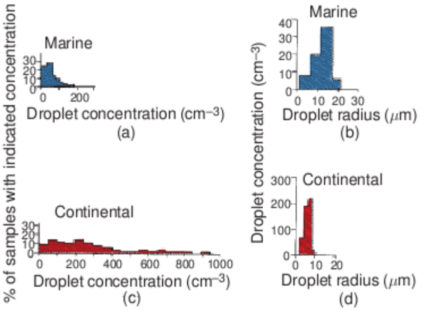


Fig. 6.7 (a) Percentage of marine cumulus clouds with indicated droplet concentrations. (b) Droplet size distributions in a marine cumulus cloud. (c) Percentage of continental cumulus clouds with indicated droplet concentrations. (d) Droplet size distributions in a continental cumulus cloud. Note change in ordinate from (b). [Adapted from P. Squires, "The microstructure and colloidal stability of warm clouds. Part I—The relation between structure and stability," *Tellus* **10**, 258 (1958). Permission from Blackwell Publishing Ltd.]

Figure from Wallace and Hobbs

Question 2:
How do the broad size distributions formed that are commonly measured?

Possible explanations

- Giant cloud condensation nuclei (GCCN)
- Turbulence
- Radiative broadening
- Stochastic collection

Stochastic collection

continuous collection model

- collector drop collides in continuous and uniform fashion with smaller cloud droplets which are uniformly distributed in space
- therefore collector drops of the same size grow at the same rate if they fall through the same cloud of droplets

stochastic (statistical) collection model

- treats collisions as individual events, distributed statistically in time and space

Stochastic collection

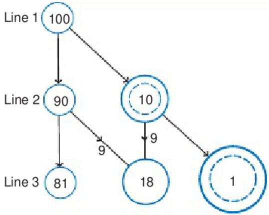


Fig. 6.24 Schematic diagram to illustrate broadening of droplet sizes by statistical collisions. [Adapted from *J. Atmos. Sci.* **24**, 689 (1967).]

Figure from Wallace and Hobbs

- Line 1: 100 droplets start at the same size
- Line 2: after some time 10 droplets have collided with other droplets
- Line 3: second collisions produce 3 sizes

Stochastical model results in broad size distributions

Statistical growth: stochastic coalescence equation

- evolution of entire droplet spectrum needs to be taken into account
- development of droplet size spectrum in time:

Stochastic coalescence equation

$$\begin{aligned}\frac{\partial}{\partial t}n(V)dV = \quad & \frac{1}{2}\int_0^V H(\delta, V')n(\delta)n(V')d\delta dV' \\ & -n(V)dV\int_0^\infty H(V_1, V)n(V_1)dV_1\end{aligned}$$

(Melzak and Hitchfeld, 1953)

- first term: growth of smaller droplets to gain volume V
- second term: all possible captures of droplets with V by larger drops, as well as capture of smaller droplets

First solution of stochastic coalescence equation

Warshaw (1968)

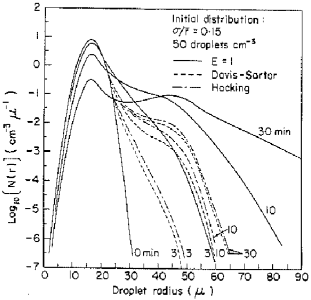


FIG. 7.10. Droplet spectrum at 3, 10 and 30 min for three different collision efficiencies. (From Warshaw, 1968.)

Figure from Rogers

- solution corresponds to average value of n(V,t) over many realizations
- n(V,t=0): Gaussian distribution
- $\sigma/\bar{r}$ = 0.15, LWC=1g/m^3
- various values for E
- fastest evolution for E=1 (geometrical sweep-out)

Development of size distribution

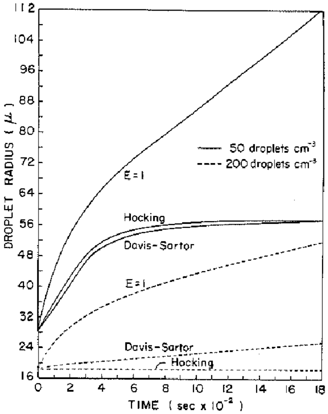


FIG. 7.11. Growth of the 100th largest droplet. (From Warshaw, 1968.)

Figure from Rogers

- radius of 100^{th} largest droplet – measure of the development of the size distribution
- initial conditions
  - 50 droplets cm^{-3} ⇒maritime cloud
  - 200 droplets cm^{-3} ⇒continental cloud
- n(V,t=0): Gaussian, $\sigma/\bar{r}$ = 0.15, LWC=1g/m^3

Statistical growth: stochastic coalescence equation

Kovetz and Olund (1969)

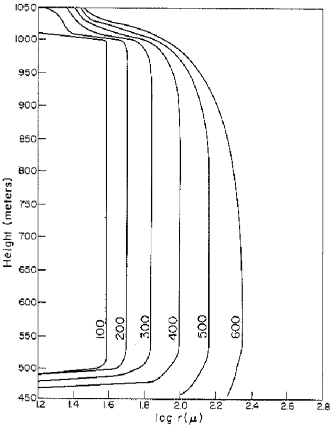


FIG. 7.12. Size of the 100th largest droplet at times indicated (sec). (From Kovetz and Olund, 1969.)

- modifications compared to Warshaw:
  - gravitational settling of droplets considered
  - condensational growth included
- initial condition as Warshaw, maritime case
- after 600s $\Rightarrow r \approx 200 \mu m$ (logr=2.3) (Warshaw only $52 \mu m$)
- faster growth when condensational growth is included

Condensation + stochastic coalescence

When condensation growth is included in stochastic growth model, droplet growth is accelerated!

Condensation growth alone narrows size distribution

Condensation + stochastic coalescence

As small droplets grow, their collision efficiency relative to large drops increase at rapid rate
$\Rightarrow$growth by coalescence accelerates

(relative velocity between collector drop and collected drop does not change due to condensational growth)

Droplet development in large convective clouds

Leighton and Rogers (1974)

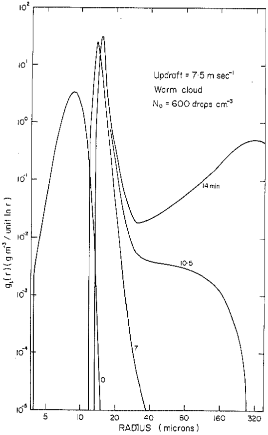


FIG. 7.13. Mass distribution after 0, 7, 10.5 and 14 min, corresponding to heights above cloud base of 0.7, 3.8, 5.3 and 7.0 km, for an updraft of 7.5 m/sec. (From Leighton and Rogers, 1974.)

Figure from Rogers

- strong updraft v=7.5 m/s
- approximations:
  - fall out of droplets neglected
  - condensation rate determined by amount of condensed water (pseudoadiabatic ascend)
- drizzle drop size reached at t$>$10.5min
- peak due to condensational growth remains stable
- without condensation $\Rightarrow$negligible growth

Growth of droplets in maritime clouds

Yang (1974)

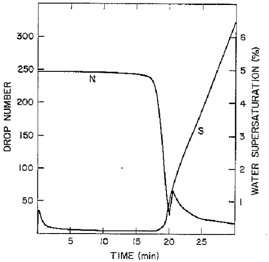


FIG. 7.14. The change with time of droplet concentration N and supersaturation S in a cloudy parcel ascending at 3 m/sec with an initial temperature of 15°C and a maritime-type spectrum of condensation nuclei. The droplet concentration is given for a mass of air which has a volume of 1 cm^3 at the initial time and is therefore proportional to the droplet mixing ratio. (From Young, 1974.)

Figure from Rogers

- calculation accounts for supersaturation and breakup of large drops
- t$<$15min mainly growth by condensation (N=const)
- t$>$15min coalescence, rapid growth:
  - N decreases
  - supersaturation increases (reduced number of droplets do not accommodate all water vapor released
  - more droplets become activated, but are quickly consumed by coalescence.

Size distribution of raindrops

Measurements of the size distribution of raindrops that reach the ground can often be fitted to the same size distribution function:

Marshall-Palmer distribution

$$N(D) = N_0 \exp -\Lambda D$$

$N(D)dD$ – number of drops per unit volume with diameters between D and $D + dD$
N_0 and Λ – empirical fitting parameters

N_0 almost const., Λ varies with rainfall rate

can be explained by breakup of large drops

Efficient solution of stochastic coalescence equation

An efficient stochastic algorithm for studying coagulation dynamics and gelation phenomena

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Abstract. A new class of efficient stochastic algorithms for the numerical treatment of coagulation processes is proposed. The algorithms are based on the introduction of fictitious jumps combined with an acceptance-rejection technique for distributions depending on particle size. The increased efficiency is demonstrated by numerical experiments. In particular, gelation phenomena are studied.

published in 2001

Formation of rain in warm clouds

Effects of stochastic coalescence and air turbulence on the size distribution of cloud droplets ☆

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Accepted 9 December 2005

Abstract

An open question in warm rain process and precipitation formation is how rain forms in warm cumulus as rapidly as it has sometimes been observed. In general, the rapid growth of cloud droplets across the size gap from 10 to 50 μm in radius has not been fully explained. Three aspects related to the air turbulence and stochastic coalescence are considered here in an attempt to resolve this open question. The first is the enhanced geometric collision rates caused by air turbulence. The second is the effect of air turbulence on collision efficiencies. The third is stochastic fluctuations and correlations in the collision–coalescence process. Rigorous approaches are developed to address these issues. Preliminary results indicate that turbulence could shorten the time for drizzle formation to about a half of the time needed for the same growth process based on hydrodynamic–gravitational mechanism alone. To address the effect of stochastic correlations, we derive and validate a true stochastic coalescence equation. It is hoped that this new mean field equation will be useful in the future to improve the deterministic kinetic collection equation.

Formation of rain in warm clouds

New Observations of Precipitation Initiation in Warm Cumulus Clouds

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(Manuscript received 27 August 2007, in final form 1 February 2008)

ABSTRACT

The mechanism responsible for formation of rain in warm clouds has been debated for over six decades. Here, the authors analyze new measurements of shallow cumulus made with a phase Doppler interferometer during the Rain in Cumulus over the Ocean (RICO) experiment. These observations show that drops sufficiently large (>55-μm diameter) to initiate precipitation (termed collision–coalescence initiators or CCIs) are found preferentially at cloud top, tend to cluster with each other, and are found in environments that are thermodynamically, dynamically, and microphysically distinct from those of smaller drops. The CCI environments exhibit cloud spectra that are shifted to larger sizes, with enhanced broadening toward larger drop sizes. Increased entrainment is also associated with CCIs, suggesting that it is an important process in CCI production. A simple model combining inhomogeneous mixing and condensation is inadequate to explain these observations. It is hypothesized that CCIs are produced in cloud-top regions where turbulence generated by entrainment mixing locally enhances collision–coalescence rates.

Formation of rain in warm clouds

Giant Sea-Salt Aerosols and Warm Rain Formation in Marine Stratocumulus

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(Manuscript received 7 September 2007, in final form 4 April 2008)

ABSTRACT

The concentrations and sizes of smaller aerosols (radius smaller than 0.5 μm) in the marine atmosphere vary owing to natural and anthropogenic factors. The concentrations and sizes of giant and ultragiant aerosols vary primarily due to wind-speed-dependent wave breaking. In climate models the formation of warm rain from marine stratocumulus clouds is usually parameterized based on the drops that form on the smaller aerosols. The present process study, using a stochastic Monte Carlo cloud model, shows that the variability of giant sea-salt aerosols and the variability of smaller aerosol cloud condensation nuclei are equally important in determining precipitation flux in marine stratocumulus. This strongly suggests that the effects of giant sea-salt aerosols should be included in the parameterization of warm rain formation in climate and other large-scale models.

The above results are based on highly detailed calculations of droplet growth in an idealized marine stratocumulus cloud; the authors believe that other marine stratus cloud conditions may change the calculated rain rates but that the conclusions regarding the relative importance of small and giant aerosols are robust.

Influence of gravity on collisions of monodispersed droplets in homogeneous isotropic turbulence

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(Received 6 August 2009; accepted 18 November 2009; published online 30 December 2009)

This paper studies the gravity influence on collisions of monodispersed droplets in homogeneous isotropic turbulence by means of direct numerical simulations (DNSs). The DNS results show that, in certain Stokes and Reynolds regimes, collision frequencies are significantly reduced in the presence of gravity. Those decreases are mainly attributable to the decrease in the droplet relative velocity, since the change in radial distribution function—often referred to preferential concentration—is small. Further analysis of the results reveals that droplet sedimentation due to gravity shortens the droplet-fluid interaction time, consequently weakening the relative motions between droplets. These observations lead to an analytical model that can be used to estimate the velocity fluctuations of sedimenting particles under gravity. Utilizing this model, we constructed a further analytical model for estimating the gravitational influence on collisions. Given flow and particle parameters, the model calculates the ratio of collision frequencies with and without the effect of gravity. Past studies simply noted that the gravitational influence is negligible when the droplet sedimenting velocity is much smaller than the flow velocity fluctuations. Our analytical model further suggests that the gravitational influence on collisions of monodispersed cloud droplets with non-negligible sedimentation rates stays negligibly small even in high Reynolds number flows, such as those typically found in convective clouds.
[doi:10.1063/1.3276906]

Formation of rain in warm clouds

Formation of rain in warm clouds still not completely understood, mainly because observed growth rates are larger than theoretical predictions !!